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SHEAR DEFLECTION OF WIDE FLANGE STEEL BEAMS IN THE PLASTIC RANGE

W. J. Hall,[1] J. M. ASCE, and N. M. Newmark,[2] M. ASCE

SYNOPSIS

The following topics are included in this paper:

1. The results of tests of two continuous steel I-beams, each beam having regions subjected to pure bending, bending combined with low shear, and bending combined with high shear.
2. A comparison of some of the test results with current design specifications.
3. Data for estimating the component of deflection caused by shear when regions of the beam have undergone general shear yielding.
4. Theoretical examples illustrating the effects of general shear yielding of the web on the deformation characteristics of beams.

INTRODUCTION

The utilization of the reserve plastic strength of steel in structural design is a subject which has merited considerable attention in recent years. The economy to be gained by so-called plastic design methods lies in the fact that for redundant or complex structures the load corresponding to a specified deflection or to collapse may be considerably greater than the load corresponding to the beginning of local inelastic action. Deflection rather than load or stress often may be the controlling factor in design.

A review of the literature indicates that up to the present time the discussions of the deflection of structures loaded beyond the elastic limit have been restricted almost exclusively to the effect of bending alone. Although there may be good reasons for such restrictions in many cases, it is important to be able to recognize when shear should be considered and to have the concepts and data available which will make it possible to calculate, or at least to estimate, the shear deflection in the plastic as well as in the elastic range.

One objective of the beam tests and studies reported in this paper was to evaluate the importance of shearing stresses and deformations on the behavior of beams, particularly in reference to load-carrying capacity. Another objective was to obtain information which would make it possible to predict the shear deflection of beam sections subjected to high shear forces. At the same time additional data were obtained which are of interest in relation to current

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specifications for allowable shear stress in the webs of beams.

Little theoretical or experimental information is reported in the literature on the plastic deformation of structural beam sections subjected to high shear forces. In England the influence of shear forces on the behavior of I sections has been investigated experimentally by Baker and Roderick [1]*, by Hendry [4], and theoretically by Horne [5]. In the United States the subject has received limited study. Johnston, Yang, and Beedle [7] point out some of the implications of shear yielding of the web. A recent report by Leth [9] is devoted to a theoretical treatment of the problem and in large part this amounts to an extension of Horne's work [5].

For the most part the foregoing studies were concerned with the reduction to be expected in the so-called fully plastic moment. Several of the studies indicated that high shear forces caused a small to practically negligible reduction in the load-carrying capacity (except for extremely short sections), while others indicated that there could be a marked reduction in the load-carrying capacity for some sections.

In the experimental portion of the investigation reported in this paper two continuous beam tests were made using one type of beam specimen. Each beam had sections subjected to pure bending, bending combined with low shear, and bending combined with high shear. The tests were carried far into the plastic range. In analyzing the data it was assumed that the bending and shear components of deflection could be separated and that their combined effect could be obtained by superposition. This approach resulted in data which can be used to estimate the shear deflection of other types of beam sections. Obviously two beam tests alone cannot provide a complete picture of the shear problem. Rather, they should be considered as exploratory tests which show what may be expected under certain conditions. Although the shear appeared to have little effect on the load-carrying capacity, it did exhibit considerable effect on the load-deflection response, a fact which is worthy of consideration in assessing the applicability of present specifications. It may be concluded from this study that the effect of high shear forces is not of great importance except for unusual loading conditions. In the last section of the paper some of these unusual loading conditions are described along with illustrations of the effects which might be expected if high shear forces were present.

Description of Beam Specimens and Apparatus

Two continuous beams of 8WF58 sections were tested in the as-rolled condition as a part of this investigation. This section was selected since it possessed excellent rotation capacity[3] and a relatively thick web which was desirable for sustaining large shear deformations.

The setup for the continuous beam tests is shown schematically in Fig. 1. The central span of 9 ft. was maintained in both tests, with the central load points being symmetrically spaced at the one-third points for beam B1 and at the one-sixth points for beam B2. Symmetrical loading was used in order to simplify analysis of the data. Unsymmetrical loading would have made the analysis extremely difficult as is illustrated by the examples presented in the last section of this paper.

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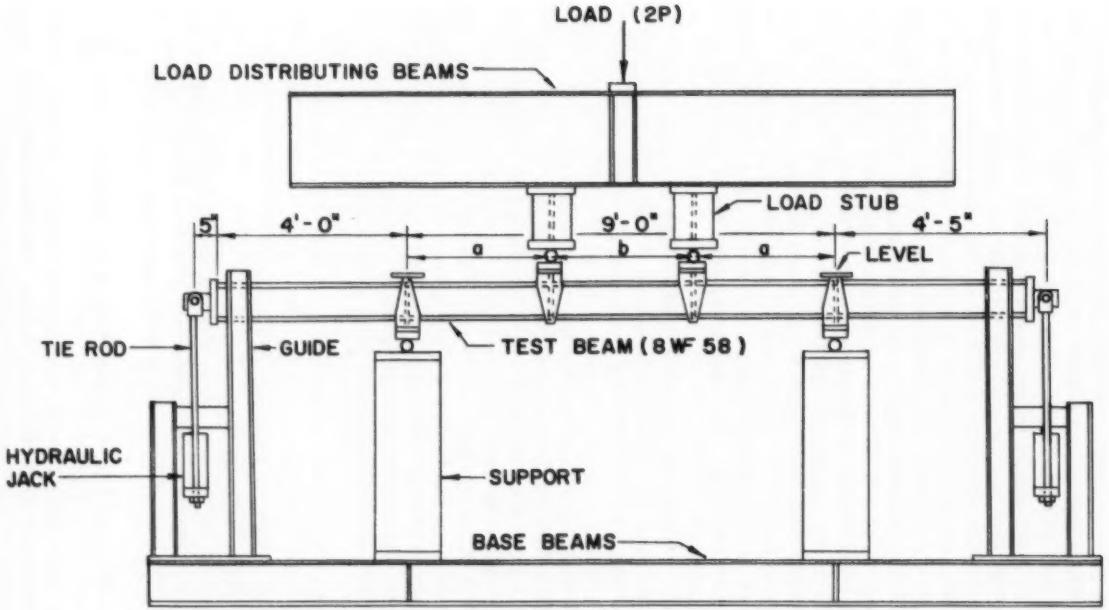
*Numbers in parentheses refer to references listed in the bibliography.

3. Rotation capacity refers to the ability of a section to sustain the "hinge" moment through the required rotation without failure by local flange or web buckling.



BEAM	a	b
B1	36"	36"
B2	18"	72"

FIG. 1 SCHEMATIC VIEW OF CONTINUOUS BEAM TEST SETUP

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Both beams were tested to simulated fixed-end beams by using the end cantilever sections to maintain zero rotation at the supports (Beams B1 and B2a)[4]. Following the testing of beam B2 as a continuous beam (B2a), the cantilevers were freed and the central portion was tested in pure bending (B2b).

The load and reaction brackets were identical, and the same type was used in both tests. The brackets were not designed to simulate any particular type of connection. To a certain extent they resembled a splice plate, although not in the true sense, since the beam was not cut to insert the plates but instead was continuous throughout its entire length. The bracket plates were slotted to fit the wide flange beam sections and were welded completely around the beam. These load and reaction brackets showed no signs of failure in either test.

The instrumentation included strain measurements made with SR-4 wire resistance gages (including post-yield gages) and mechanical gages. Deflection readings were made at selected points along the beam with reference to a line through the reaction. Whitewash was applied to more than half the beam in each test to indicate the yielding pattern and progression of yield zones. As an extension to the SR-4 rosette gage readings in the web, measurements of shear detrusion on the shear-moment section were made by using direct reading mechanical gages and gage holes on a 2 in. square spacing. These gage holes were placed symmetrically about the longitudinal axis of the web and on both sides of the web. The level mounted on top of each reaction bracket was used to maintain the endfixity during the test. The distribution of load between the load points was checked by two instrumented "load stubs" which were placed between the distribution beam and the rollers. The cantilever ends were loaded by jacks through instrumented tie rods.

The section properties of beams B1 and B2 are presented in Table 1 and the results of the tension coupon tests are summarized in Table 2. Longitudinal residual strain measurements were made on a portion of the piece from which beam B1 was cut, and showed a maximum residual strain of 0.00045 in. per in. at the center of the web, or about 40 per cent of the yield point strain.

The testing procedure consisted of loading with the 3,000,000-lb. machine while at the same time jacking down the ends of the cantilever to keep the level bubbles at the reaction points centered. When yielding occurs under constant load in the plastic range it takes considerable time for the beam to come to rest. Under such conditions, when the beam did not come to rest in 15 to 20 minutes, the load was decreased slightly (one to not more than two kips in a total load of 200 to 400 kips) to stop the beam from deforming while readings were taken. In this manner it was possible to run the tests in 14 to 16 hours.

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4. The beam descriptions used hereafter are as follows:
   - B1 refers to the first continuous beam test.
   - B2 refers to the second continuous beam when no distinction is required regarding the two separate tests performed with this beam.
   - B2a refers to the first test of beam B2 in which shearing action was studied
   - B2b refers to the subsequent pure bending test of the central portion of beam B2.

Each beam had sections under pure bending (the central section), bending combined with low shear (cantilever sections), and bending combined with high shear (the shear-moment sections or V-M sections). The latter refers to the portions between the central load points and the reactions.

Results and Interpretation of Tests

General Remarks

Neither of the two continuous beams had reached its maximum load by the end of the test. The test of beam B1 was stopped at a load of 269 kips because of a jack failure. At that time the compression flanges inside each reaction, i.e., in the shear-moment section, were just starting to buckle. This buckling was barely visible to the eye. Beam test B2a was carried to a deflection which was thought to be sufficient to obtain the desired data, but at the same time was stopped while the central portion was still in the elastic range. This was done in order to allow the central portion to be used in a pure bending test.

The pure bending test (B2b), which consisted of freeing the cantilever ends and testing the central portion as a simple beam, was made in order to obtain an experimental moment-curvature relationship for comparison with that obtained from the tension test coupon data. Moreover, it was necessary to have fairly reliable moment-curvature information for the analysis of the shear-moment data. The central section failed by gradual lateral buckling as the test was carried into the strain hardening range.

Deflection calculations were made by using the numerical integration procedure developed by N. M. Newmark [12]. This and other procedures for computing flexural deflections of beams in the plastic range are discussed in reference [8]. The procedures described in the latter article employ assumptions as to the propagation of the plastic regions and modifications of the moment-curvature diagram which tend to simplify the calculations but which yield less accurate results. For many applications the simpler procedures are preferable, but for applications where high accuracy is needed the numeric al integration procedure is the most desirable.

The separate deformation concepts attributed to moment and shear are illustrated in Fig. 2. The basic hypothesis used in the reduction of the moment-shear data is that the bending and shear behavior can be separated and treated independently in both the elastic and plastic range. In other words, superposition is assumed to hold through the range of response under study. This appears to be the best method of handling the two effects. It is believed that when the data obtained from these tests are used to compute the shear deflection of other similar beams, the predicted deflections will be reasonably accurate. Data for making an estimate of shear deflections when shear forces are high have not previously been available in the literature.

The following discussion of the test results is presented in the order in which the analyses were made, i.e., pure bending test, cantilever sections (bending combined with low shear) and V-M sections (bending combined with high shear). This is followed by a discussion of certain phases of the test results in the light of current design specifications, and by a presentation of data for predicting the shear deflection of beams after general shear yielding has occurred.

Pure Bending

The simple plastic theory of pure bending of mild steel beams is based on three assumptions:

a. The flexural strain is distributed linearly over the cross-section of the beam, as in the elastic bending theory.

b. The stress-strain relationship in a bent beam is the same as that determined from a static tension test of a coupon of the material.

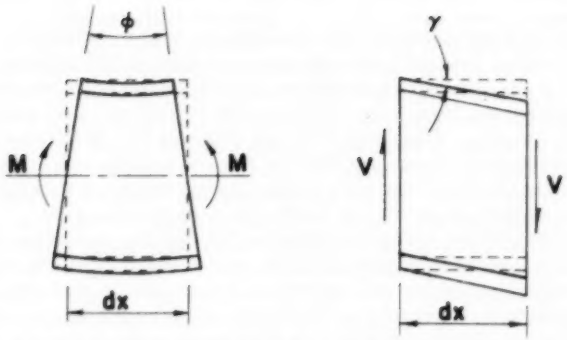


FIG. 2 DEFORMATION OF A BEAM SECTION DUE TO BENDING MOMENT AND SHEAR

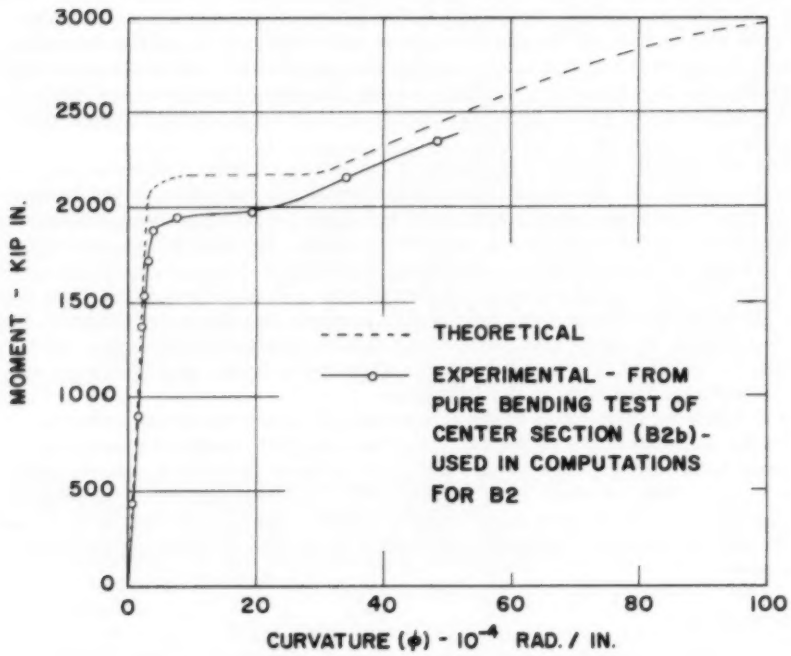


FIG. 3 MOMENT-CURVATURE DIAGRAM FOR BEAM B2

c. The stress-strain relationship is the same in compression as it is in tension.

In the elastic range the moment M and the curvature ϕ of a bent beam are related by the following expression, in terms of the modulus of elasticity E, and the moment of inertia I:

$$\phi = M/EI$$

In the plastic range the moment-curvature relationship is obtained from a consideration of the stress and strain conditions assumed to exist in a section of the member. The calculation of the moment requires a consideration of the stress block assumed to be acting on the section. Since the strain is distributed linearly, the stress block corresponding to a given distribution is actually the stress-strain curve (up to the maximum strain) with the strain axis adjusted to one-half the beam depth. For a given strain distribution and the corresponding stress block, the curvature and moment can be calculated. Methods of calculating the moment-curvature (M- ϕ) diagram have been presented in many articles (3, 10).

The M- ϕ diagram obtained from the pure bending test (B2b) is presented in Fig. 3. The experimental plot was obtained by a curve-fitting process in which the curvatures were computed so as to make the derived deflections check the measured deflections. A check was obtained by computing the curvatures from the strain gage readings. An independent check on the curve-fitting process was obtained by using the measured deflections and a second order difference equation. All the latter values fell so close to the original curve that they could not be distinguished with the scale used in Fig. 3.

The initial slope of the experimental M- ϕ curve in Fig. 3 corresponds to an EI value of 69.3×10^5 ksi. With I computed to be 232 in.^{4}, E is 29.9×10^3 ksi, which is a reasonable value.

It is noted that the experimental curve falls below the theoretical curve (computed from tension coupon data) through the flat portion and then climbs again toward the theoretical curve in the strain hardening region. The decrease amounts to about 9 per cent of the theoretical value in the flat portion and about 3-1/2 per cent in the strain hardening region. This follows observations of M- ϕ diagrams made in other publications (10 and 13) which showed decreases of the same order of magnitude. The experimental M- ϕ diagram obtained from beam test B2b was used in all computations for beam B2. An adjusted curve using the same 9 per cent and 3-1/2 per cent reduction values was used in the computations for beam B1.

A moment-strain curve from beam B2b is shown in Fig. 4. The strains shown are the average of the top and bottom strains for the section. The curve could be converted to a M- ϕ diagram by dividing the strain values by one-half the depth of the beam.

The measured load versus center deflection curve for beam B2b is presented in Fig. 5. Also shown in the same figure is the load deflection curve corresponding to the theoretical M- ϕ diagram of Fig. 3. It is observed that a slight variation in the M- ϕ diagram for regions of pure bending can cause considerable differences in deflections. This is particularly true for long spans under pure bending.

Combined Bending and Shear Sections

Since both continuous beams were loaded symmetrically, little or no shear was present in the central sections during the tests. The cantilever sections

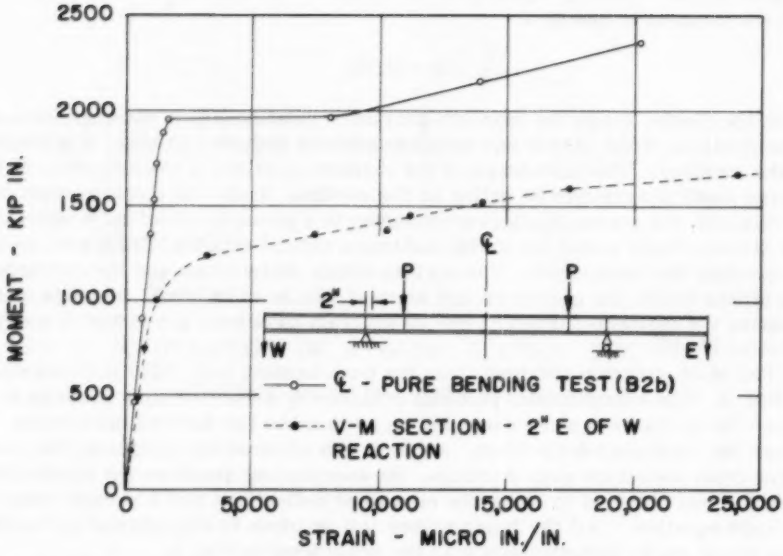


FIG. 4 MOMENT VERSUS STRAIN FOR CENTER AND V-M SECTIONS OF BEAM B2

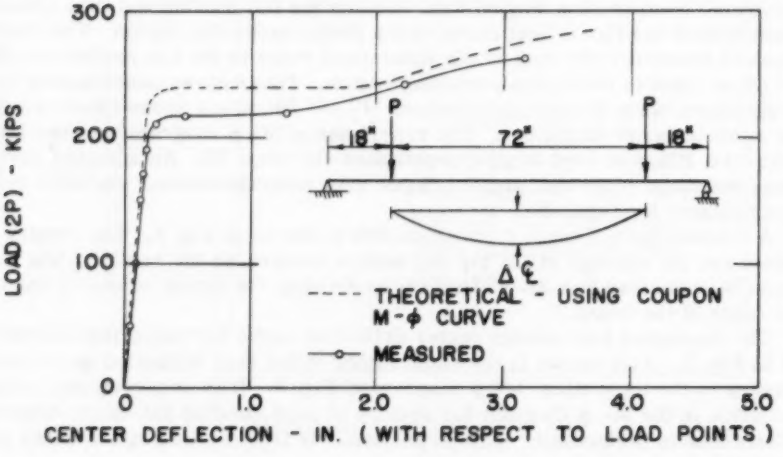


FIG. 5 LOAD VERSUS CENTER DEFLECTION FOR BEAM B2b (PURE BENDING TEST OF CENTER SECTION)

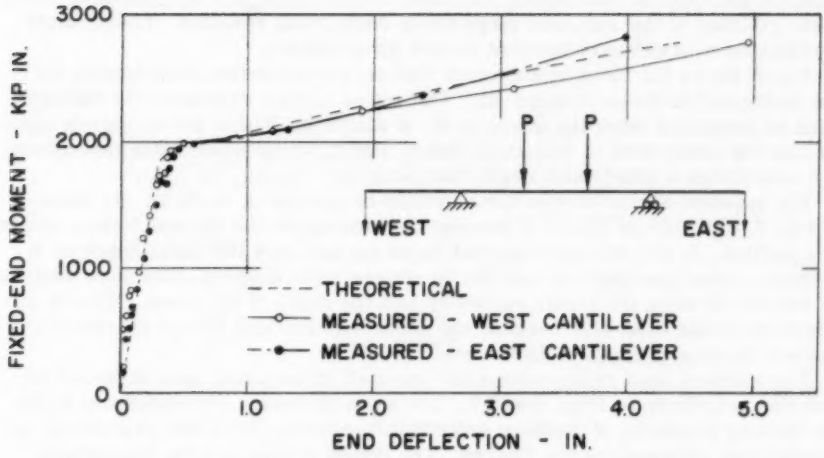


FIG. 6 FIXED-END MOMENT VERSUS END DEFLECTION FOR CANTILEVER SECTIONS OF BEAM B1

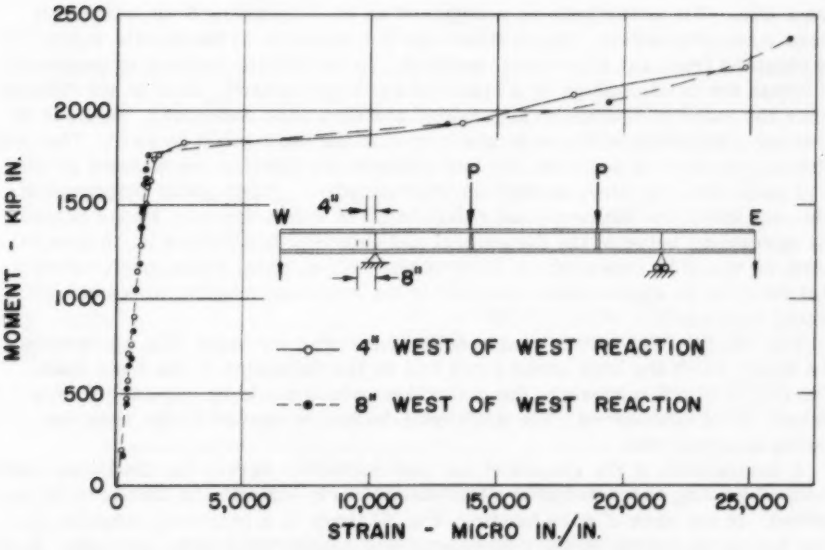


FIG. 7 MOMENT VERSUS STRAIN FOR WEST CANTILEVER SECTION OF BEAM B1

which were used to maintain the end-fixity of the main span were of such a length that the shear was relatively low in them. Shear yielding did not occur in the web of the cantilever sections in either test. In the regions between the reaction and central load points, the shear forces were of a magnitude to cause shear yielding of the web, and large shear deflections resulted. These latter portions were of primary interest in this investigation.

Fig. 6 shows the fixed-end moment versus end deflection relationship for the cantilever sections of beam B1. The dotted curves represent the deflections as computed from the adjusted M- ϕ diagram. These dotted curves also include the component of deflection due to shear, which was elastic throughout and constitutes a practically negligible quantity.

The moment-strain curves for a cantilever section of beam B1 are shown in Fig. 7. The strain values represent the average of the top and bottom strains at a section. It will be observed that these curves have the same shape as a moment-curvature diagram and can be converted to moment-curvature diagrams by merely dividing the strain values by half the depth of the beam. This is admissible in this case only because the shear was low and did not markedly influence the linear strain distribution.

The moment-load relationships for one-half of the main span of beams B1 and B2a are shown in Figs. 8 and 9. The moment values corresponding to the theoretical beginning of yielding and strain hardening (from the adjusted M- ϕ curves) are indicated on the figures. The dotted curves are the theoretical moment-load relationships which would be expected from considerations of flexure alone. The method of computing these theoretical relationships is of interest and will be described briefly. In the elastic range, the ratio between the end moment and the load point moment was 2:1 for beam B1 and 5:1 for beam B2a. The latter ratio is recognized as corresponding to an extremely steep moment gradient. Expressions for the moments in the elastic range can be obtained from any structural handbook. In the plastic range it is necessary to obtain the relationships by a trial and error procedure. This is not difficult since the beam is symmetrically loaded and thus once redundant. Because of symmetry the slope at the ends and center of the beam must be zero. The procedure consisted of selecting the end moment and altering the moment at the load point until the slope conditions were satisfied. Since shear deformation does not affect the moment-load relationship in symmetrically loaded beams, the agreement between the theoretical and experimental curves is, in general, good, as would be expected. A study of the load-moment relationship reveals that there is no appreciable reduction in the resisting moment which can be traced to shear.

Fig. 10, the load versus center deflection curve for beam B2a, illustrates the effect which the high shear force had on the deflection of the main span. The dotted curve represents the deflections which would be expected if only flexure were considered. The additional deflection caused by the shearing action is noteworthy.

A comparison of the shapes of the load deflection curves for the three conditions of loading encountered in each beam clearly indicate the trends to be expected. In the case of pure bending (Fig. 5) there is a relatively long flat portion before strain hardening commences and causes the load to increase. In the cantilever sections (Fig. 6), which have a moment gradient but low shear forces, i.e., bending is still the predominant action, strain hardening commences quickly and causes the curve to rise almost immediately after yielding has occurred at the fixed end. For the region with a steep moment gradient and corresponding high shear forces, general shear yielding of the web causes a break in the load deflection curve (Fig. 10) which may be at a much lower load than that

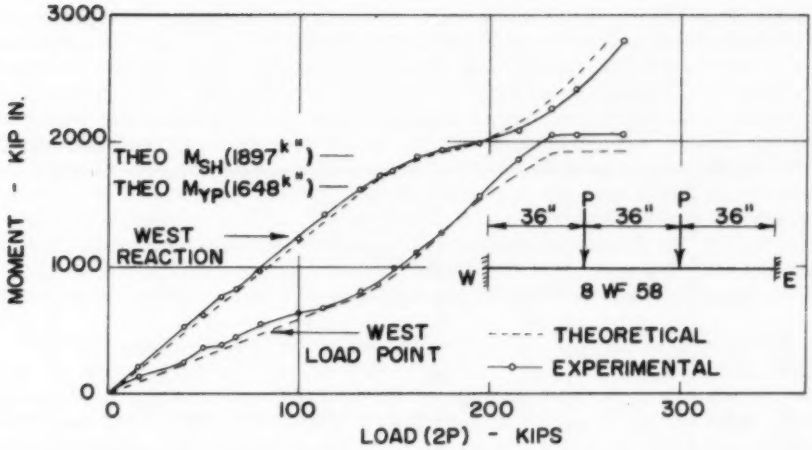


FIG. 8 MOMENT-LOAD RELATIONSHIPS FOR BEAM B1 (WEST END)

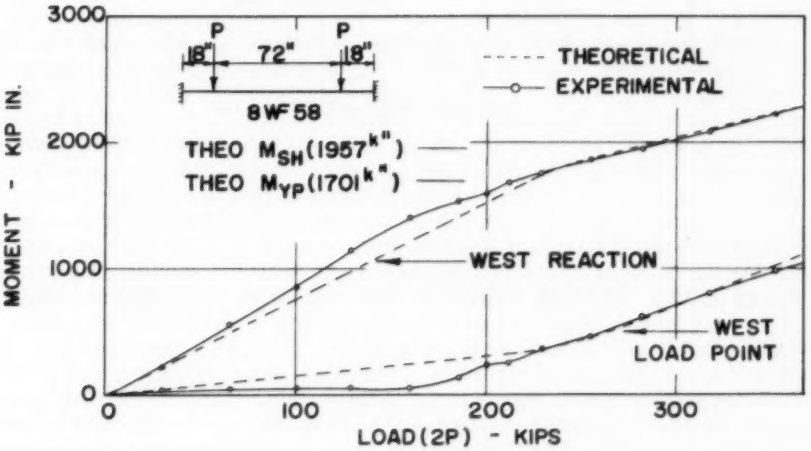


FIG. 9 MOMENT-LOAD RELATIONSHIPS FOR BEAM B2a (WEST END)

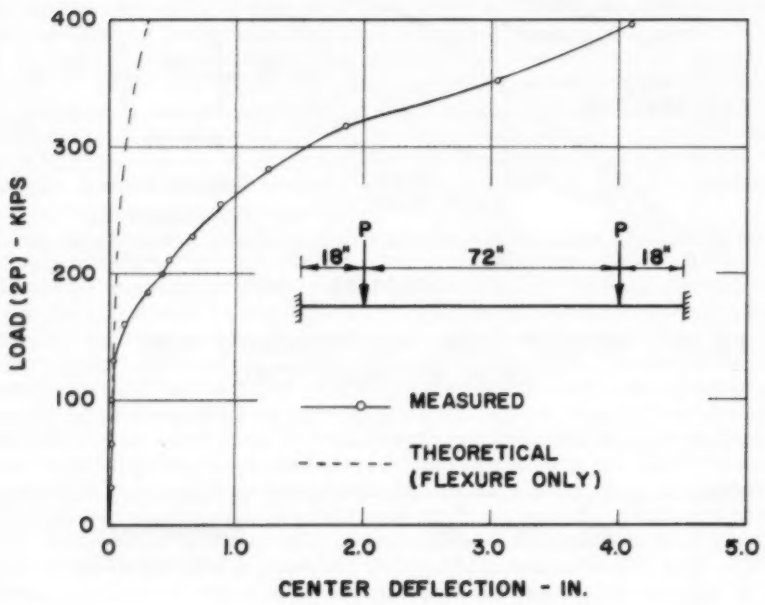


FIG. 10 LOAD VERSUS CENTER DEFLECTION FOR BEAM B2a

which would be expected from flexural yielding alone.

The moment-strain relationship at a section in the V-M region of beam B2 is presented in Fig. 4. The strain values are the average of the top and bottom surface strains. This particular moment-strain curve was presented in order that it might be compared with one from a pure bending section (Fig. 4) and a cantilever section (Fig. 7). General shear yielding of the web markedly influences the strains in the flanges and actually such moment-strain curves have little significance. They should not be interpreted as an equivalent moment-curvature relationship for the shear-moment section since a study of other strain data taken in this region shows a different shaped curve to exist for each section; however, the load corresponding to the break over in any moment-strain curve in the V-M section corresponds identically to the load at the sharp break in the load-deflection curve. The longitudinal strain distribution in the shear-moment sections is extremely erratic and emphasizes the fact that the strain distribution is extremely complicated in short sections. The boundary conditions and complex residual strain pattern markedly influence the strain distribution.

The deformed shapes of the beams, as well as the test layout, are shown in Figs. 11 and 12. Fig. 12 clearly illustrates the shear deformation. The central portion (to the left in Fig. 12) is still in the elastic state and it was this portion which was later tested separately in pure bending (B2b). Close-up views of the cracked whitewash patterns in the shear-moment sections at two different loads are shown in Figs. 13 and 14. Flaking of the whitewash was initially observed at a load of 50.5 kips in beam B1 (shear = 24.2 kips)[5] and 30 kips in beam B2a (shear = 15.6 kips)[6] at both the reaction and load ends of the V-M section in the web corners subjected to compressive flexural stress. This is to be expected since the residual strains in the web are of a compressive nature. The yield patterns were noted to be almost identically symmetrical on either side of the web.

Methods of estimating the elastic shear deflection of beams are well known. Fife and Wilbur (2) summarize the test data from some simple beam tests. They conclude that the nominal shear stress value which should be used in the shear strain computation is obtained by dividing the shear by the gross web area (the web thickness times the depth of the beam). This value of shear stress divided by the shearing modulus of elasticity (G) gives the shear strain. The shear strain, which is in units of in./in., multiplied by the length in question will give the component of deflection due to shear. This is the conventional method of computing the elastic shear deflections of structural beams and gives a reasonable answer. Many refinements can be made by considering more accurate stress distributions, and these are presented in many strength of materials and theory of elasticity textbooks.

One of the objectives of this investigation was to obtain data by means of which an estimate of shearing deflection could be made for wide flange beams loaded into the plastic range. In order to accomplish this aim, several methods were employed to obtain a shear versus shear strain relationship which could be used to predict shear deflections. One method of obtaining a shear versus shear strain curve consisted of (1) computing the deflections due to flexure alone, (2) subtracting these deflections from the measured deflections, and (3)

5. These shears are slightly different from one-half the load because of slight dissymmetry in the position of loading and in the beam properties.
6. Same as 5.

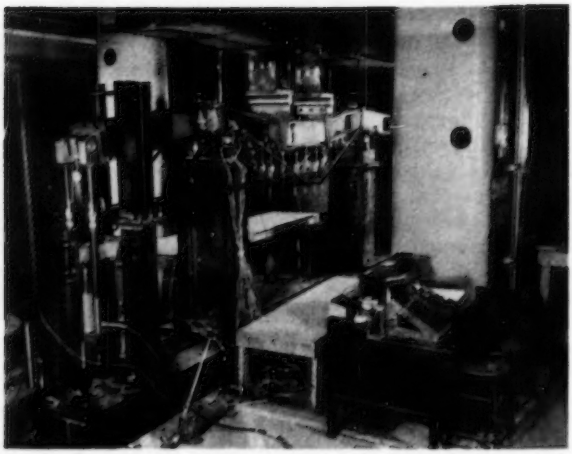


FIG. II BEAM B1 AFTER TEST

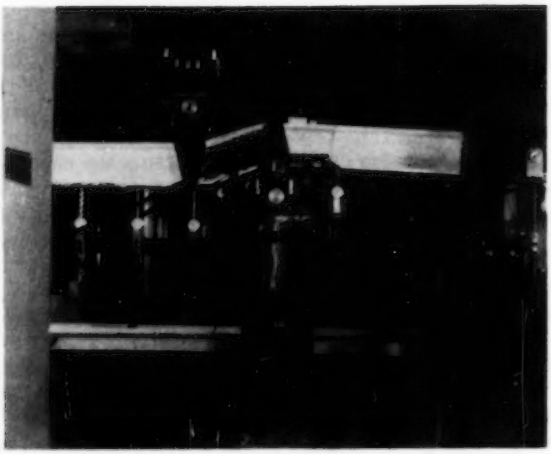


FIG. 12 SHEAR DEFORMATION OF BEAM B2a(AFTER TEST)

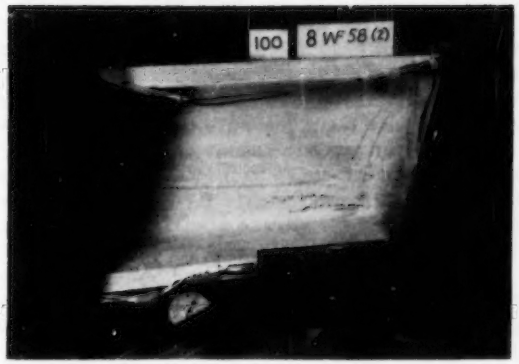


FIG. 13 V-M SECTION OF BEAM B2a - LOAD = 100 KIPS
(REACTION ON RIGHT)

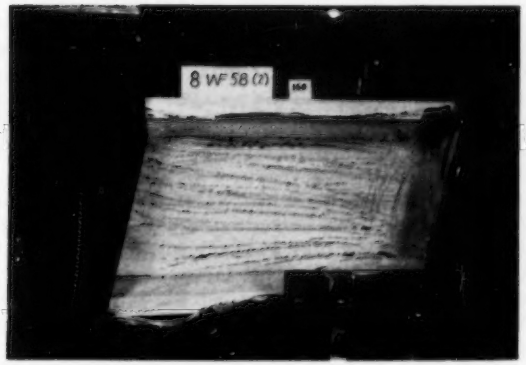


FIG. 14 V-M SECTION OF BEAM B2a - LOAD = 160 KIPS
(REACTION ON RIGHT)

taking the difference as due to shear. These differences in deflection were divided by the length between the deflection points to obtain a measure of the shear strain. Shear strain values were also computed from a number of rosette gage sets which were mounted symmetrically on both sides of the web of the beam. For a given symmetrical pair of rosette gages the strain readings for corresponding arms were averaged and these were used in the rosette strain analysis. In order to extend the range of the shear strain data obtained from the rosette gages a series of gage holes were placed on both sides of the web. From measurements of the lengths of the sides and diagonals of the square gage grids at various loads, the angle changes in the square grids were computed by trigonometry. Since the grids were symmetrical about the mid-line of the web, the angle changes were the shear deformations.

A representative illustration of the data obtained from the deflection procedure is presented in Fig. 15, from the grid layout in Fig. 16, and from the rosette gages in Fig. 17. The shear versus shear strain diagrams obtained by the three methods show excellent agreement even when plotted to a larger scale.

SR-4 strain gage rosettes which were placed on the web of the beam primarily to obtain shear strain data in the plastic range also provide some information on the elastic shear stress distribution. The rosette gage shear stress values are tabulated in Table 3 along with the VQ/Ib and V/A_w values.[7] As expected, the greatest discrepancy between the rosette and theoretical shear stress values occurs for those rosettes nearest the reaction or load points.

The foregoing data have a bearing on current design practice and the following discussion will attempt to point out some of the more interesting aspects. When the first shear yielding was noted in the whitewashed portion of the web of beam B1 at a load of 50.5 kips, the maximum shear stress for rosette gage set No. 3 was computed to be 8,900 psi. The nominal shear stress is seen to be 6,350 psi from Table 3. For beam B2a the corresponding values were 3,740 psi for rosette set No. 6 at a load of 30 kips and a nominal shear stress value of 3,500 psi. The important thing to note in this case is that the appearance of yield lines at such low stresses does not mean that structural failure is iminent. The complex residual stress field in the web tends to initiate this early yielding. The danger point is reached when a general shear yielding of the web has occurred and there is a corresponding sharp break in the load deflection diagram (Fig. 10). This break-over corresponded to a shear of about 69 kips in both beams. Based on the gross web area as defined in the specifications, this amounts to 15,000 psi for the as measured sections. With no corrections for the tension yield point values, since they are only slightly above 33,000 psi, a 15,000 psi shear stress provides a safety factor of only 1.15 based on the current AISC specification which calls for 13,000 psi maximum on the gross web area. This low value is of interest and requires serious consideration.

The implications of this sharp break-over in the load-deflection curve which results from general shear yielding were pointed out by Johnston in 1938 (6). Actually shear yielding may have a more direct bearing on the failure of a beam than flexural yielding under a concentrated load since in the former case a much greater portion of the beam is affected than in the latter case.

7. A_w is the web area taken from center to center of flanges (see discussion of shear stress-strain curve for a more complete description); V is the shear, Q the static moment about the neutral axis of the area above the section where the shearing stress is desired; and b is the width of the section where the shearing stress is computed.

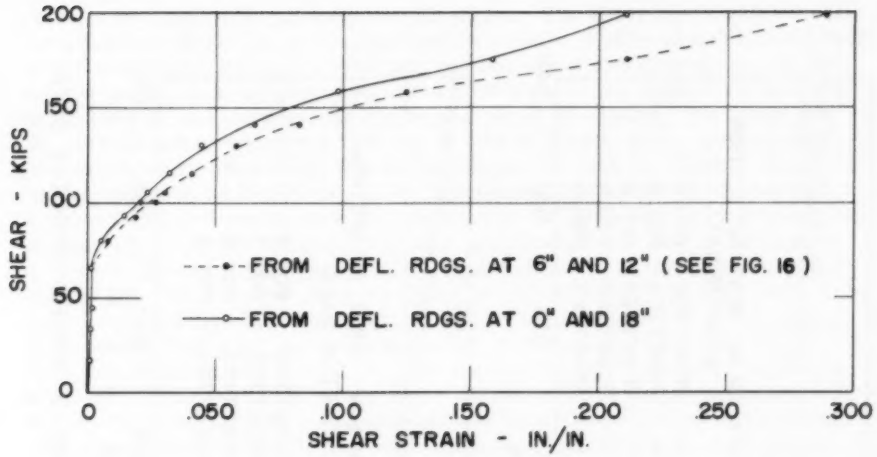


FIG. 15 SHEAR VERSUS SHEAR STRAIN FOR WEST V-M SECTION OF BEAM B2a

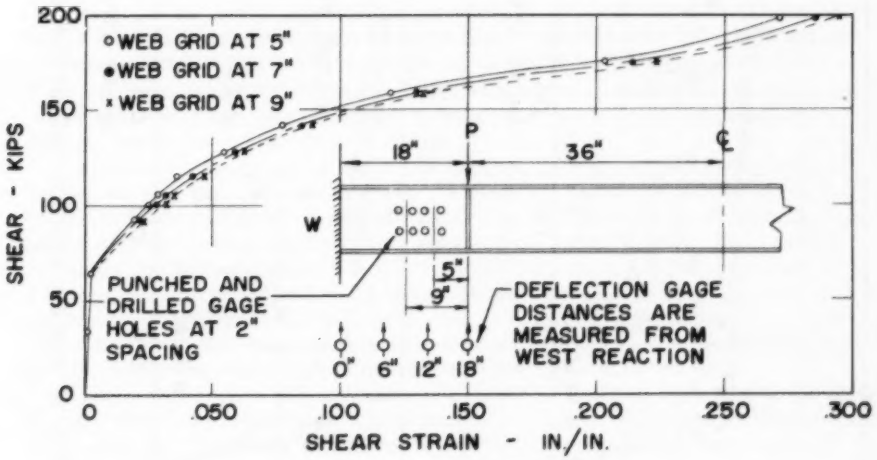


FIG. 16 SHEAR VERSUS SHEAR STRAIN FOR WEST V-M SECTION OF BEAM B2a

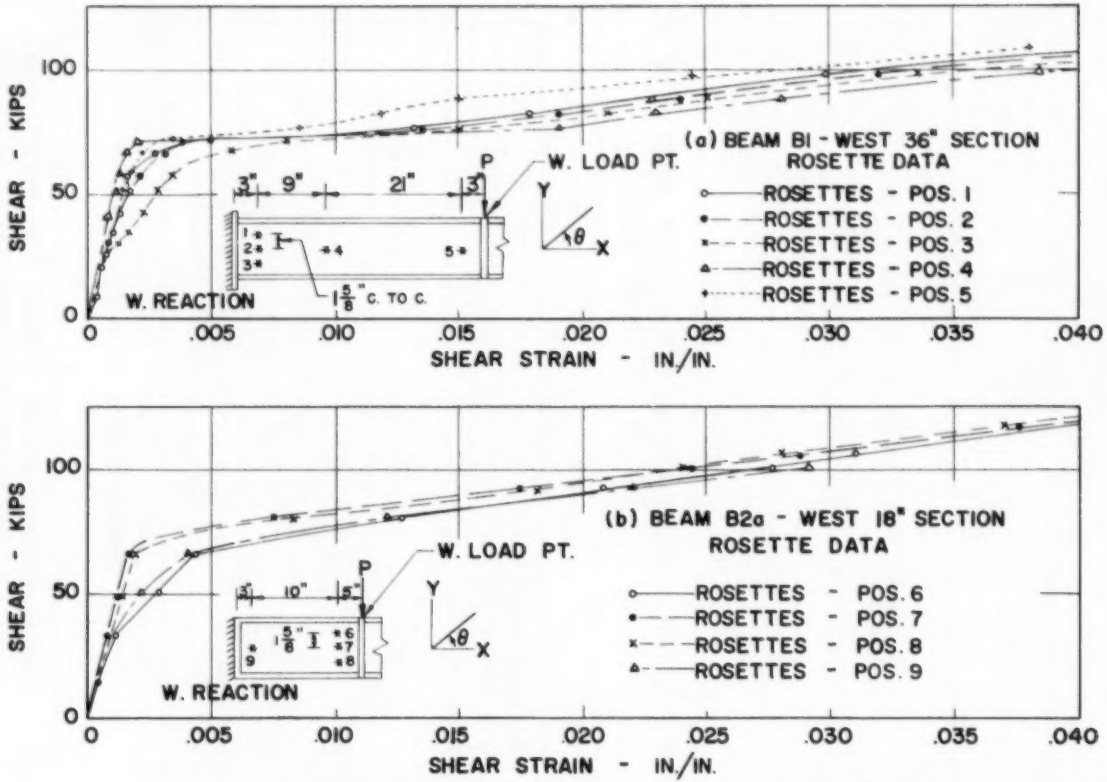


FIG. 17 SHEAR VERSUS SHEAR STRAIN FOR BEAMS B1 AND B2a (ROSETTE DATA)

Another item relating to current specifications which is of interest is the 24,000 psi stress allowed on extreme fibers in the AISC specifications for beams which are continuous at interior supports. This is a step in the direction of pseudo plastic design, but unlike the AREA and AASHO specifications there is no limit on the principal direct stress. The latter specifications call for a limit of 18,000 psi in diagonal tension in webs of girders and rolled beams at sections where maximum shear and bending occur simultaneously. In the case of beam B1, at a flexural stress of 24,000 psi at the support the moment is 1287 kip in. and the shear is 53.6 kips in the V-M section. For beam B2a the corresponding values are 1263 kip in. and 84.3 kips. In the former case general shear yielding of the web had not occurred when the flexural stress was computed to be 24,000 psi; in the latter case general shear yielding had occurred. Thus it can be seen that the position and type of loads must be given serious consideration when applying this specification. It may be dangerous to use the allowed stress value of 24,000 psi unless the shear stress is adequately low. On the other hand the AREA and AASHO specifications prove to be more conservative. For a principal direct stress of 18,000 psi at the support at the flange-web junction, the moment and corresponding shear are computed to be 920 kip in. and 38.3 kips for beam B1 and 740 kip in. and 49.3 kips for beam B2. This provides a safety factor against general shear yielding of 1.80 in the first case and 1.40 in the second case. It will be recalled that the theoretical and measured moments and loads showed close agreement for these tests (Figs. 8 and 9) so that the computed moments and shears cited above are close to those actually measured.

Shear Stress-Strain Curve

To be able to use the shear versus strain data for predicting the shear deflections of other beam shapes, it is expedient to express the shear values in terms of the average shear stress in the web. The web area used in this conversion is denoted A_w, and is equal to w(h-t) where w is the web thickness. This web area, which corresponds to the web thickness times the distance between centers of the flanges, is felt to be a reasonable value, applicable to shallow beams as well as to deep beams.

From the information amassed in both beam tests a representative shear versus shear strain curve was drawn. A study of this curve showed three rather definite regions: the initial elastic portion; after the break-over a gently sloping portion extending to a shear strain value of about 0.020 in. per in.; and a remaining portion which resembled the strain hardening portion of a tension stress-strain curve. A curve-fitting scheme was used to obtain expressions for the three portions of the curve in terms of the shear V and the shear strain γ. The expressions were then divided by A_w for the 8WF58 section to reduce the shear values to nominal shear stress in the web.

A mean shear value at the break-over in the V-γ curves appeared to be about 69 kips, which corresponds to a hear stress of 16,500 psi for the 8WF59 section. For a nominal yield point of 32.3 ksi which corresponds to the experimental M-ϕ curve of beam test B2b, a shear stress of 16,500 psi represents 0.51 σ_{yp}. These values of shear stress are nominal values based on uniform distribution of shear stress across the web; nevertheless, they fall near the range of values reported by Lyse and Godfrey (11). These investigators made a number of torsion tests on specimens taken from the webs of structural beams and found that the ratio of yield point in shear to yield point in tension varied from 0.505 to 0.634.

The curve-fitting process yielded the following expressions:

$$\gamma = \tau/G = 8.70\ V/A_w \times 10^{-5} \qquad 0 < \gamma < .00143 \text{ in./in.}$$
$$0 < \tau < 16.5 \text{ ksi}$$

$$\gamma = \frac{\tau - 16.06}{307} \qquad .00143 < \gamma < .020$$
$$16.5 < \tau < 22.2$$

$$\gamma = \left(\frac{\tau}{70}\right)^{3.40} \qquad .020 < \gamma < .300$$
$$22.2 < \tau < 49.1$$

The curve based on these expressions is presented in Fig. 18. It must be remembered that this diagram is based only on data obtained from the 8WF58 section. However, unless other specific data are available from tests, this relationship may be used to make estimates of the shear deflections of other mild steel wide flange sections.

To estimate the shear deflection of a beam it is first necessary to compute τ, which is equal to V/A_w. Enter the curve with τ and find γ, the shearing strain. The shearing strain multiplied by the increment in length along the beam gives the component of deflection due to shear over that length. This procedure has to be modified in special cases. Some of these are discussed in the next section.

Shear Deformation and Its Effect on Beam Behavior

Theoretical Examples

The purpose of this section is to illustrate some of the effects which shear deformation may have on the behavior of a beam. For symmetrically loaded beams with symmetrical boundary conditions, shear causes no readjustment of moment and slope, even in the plastic range. This was the case for the continuous beam tests which are reported in this paper. However, for unsymmetrically loaded beams, there is a readjustment of moment and slope. Some of the problems which may be expected to arise in the actual testing or analysis of an unsymmetrically loaded beam are illustrated by two examples which follow.

A 12WF120 beam having a span of 20 ft. with the load 4 ft. from one end was selected. The beam is treated first as a simple beam and second as a beam fixed at both ends. In each case bending alone and bending and shear combined are presented for comparison.

Several assumptions were made in order to simplify the analyses. The M-ϕ curve which is used in the analysis assumes elastic behavior up to the fully plastic moment value of 6150 kip in. and then becomes flat. In the first example strain hardening is considered and in the second example it is neglected. The inclusion of strain hardening would tend to reduce the deflections from those shown in the second example. Neglecting strain hardening also allows the plastic hinge method of computing deflections to be used [8] which provides a further simplification in the calculations. In the plastic hinge method the beam has elastic regions and localized plastic hinges only.

The shear stress-strain curve is taken from Fig. 18 with the exception that the second linear portion is extended beyond 0.020 in./in. to a shear strain of about 0.045 in./in. This approximation simplifies the calculations. For the following examples the deflection at B is computed. This is not the maximum

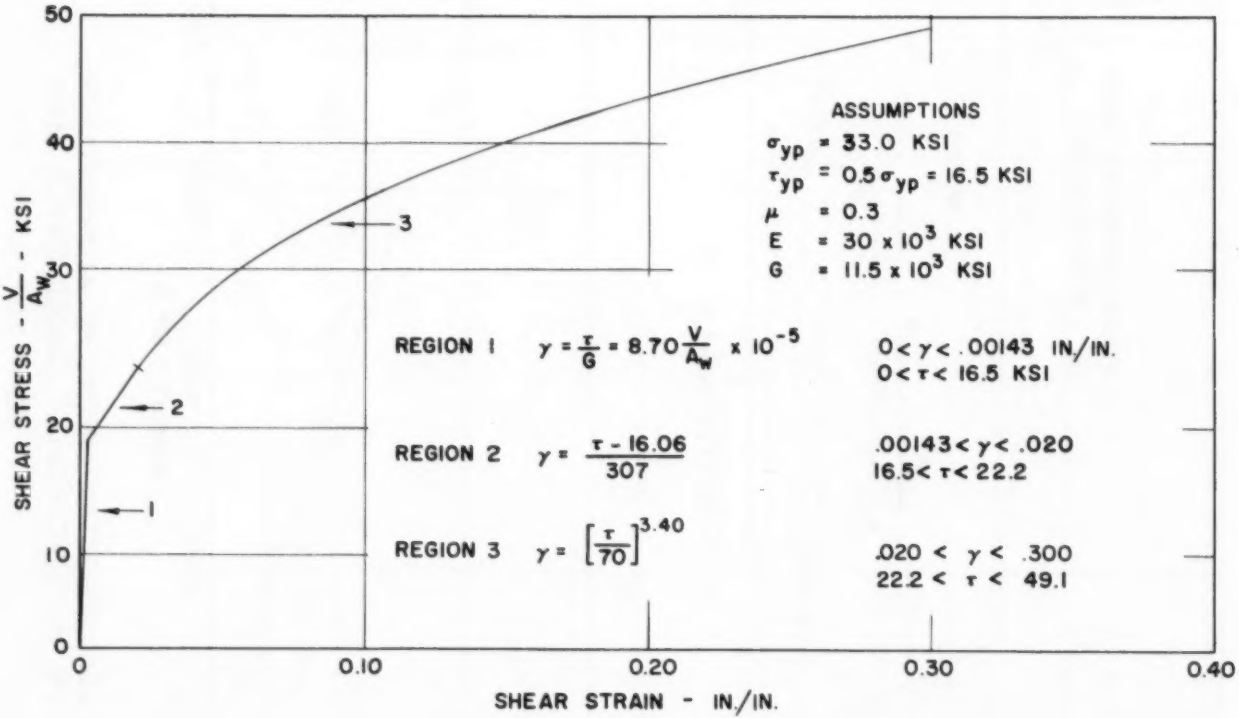
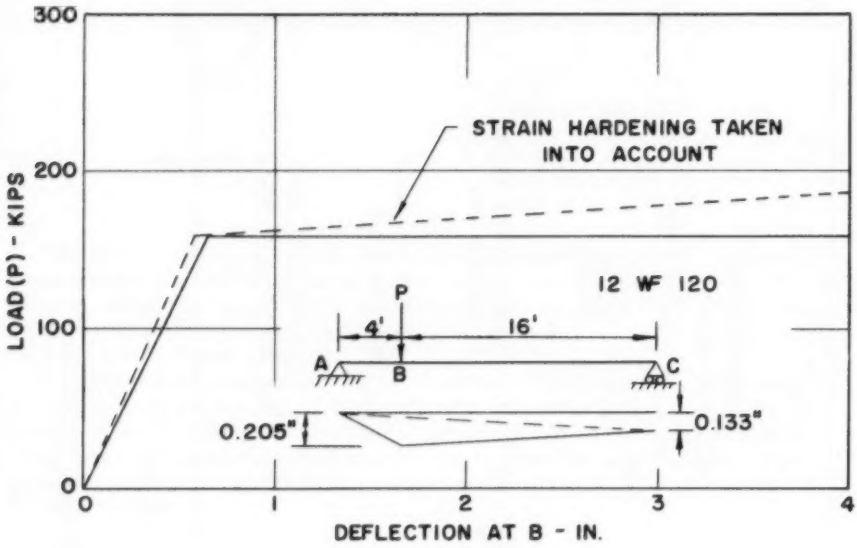


FIG. 18 SHEAR STRESS-STRAIN DIAGRAM



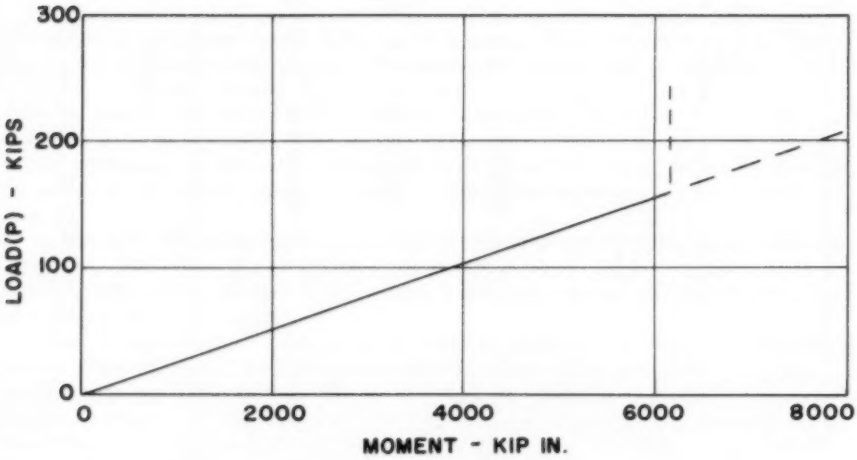


FIG. 19 LOAD-DEFLECTION-MOMENT RELATIONSHIPS FOR AN UNSYMMETRICALLY LOADED SIMPLE BEAM

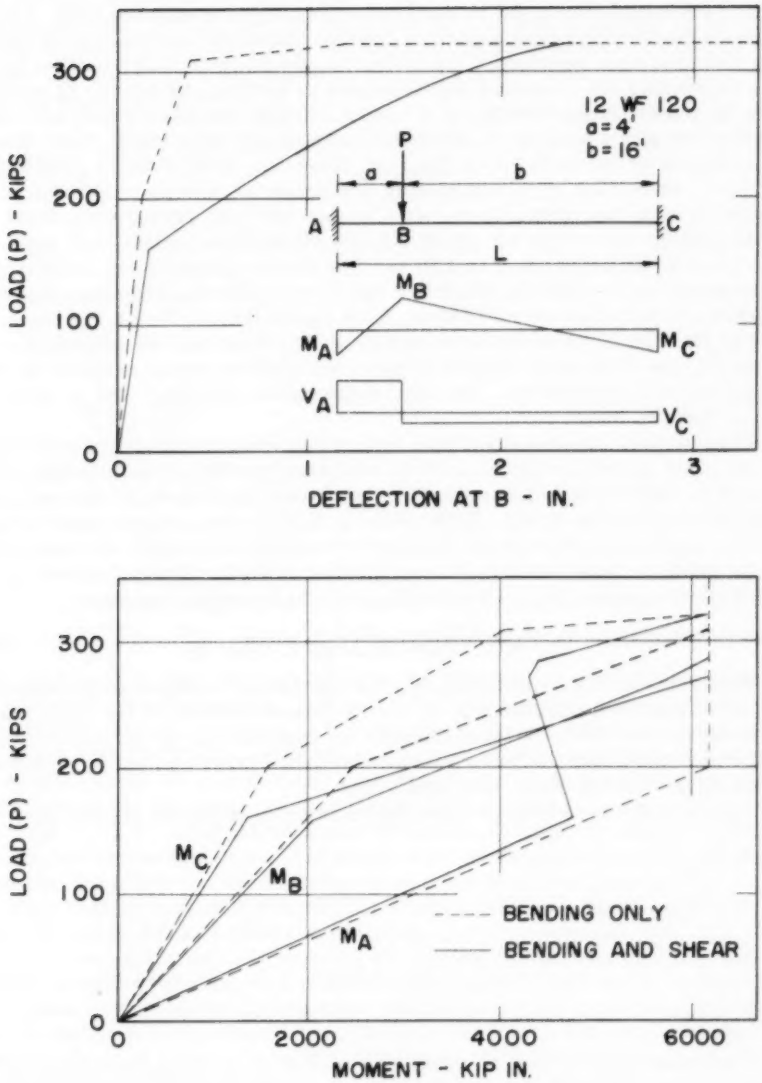


FIG. 20 LOAD-DEFLECTION-MOMENT RELATIONSHIPS FOR AN UNSYMMETRICALLY LOADED BEAM WITH BOTH ENDS FIXED

beam deflection but is near enough in magnitude to the latter to illustrate the trend.

a. Simple Beam (Fig. 19)

For the simple beam the moment increases linearly with the load. The beam remains elastic (both with regard to bending and shear) up to a load of 160 kips. The dotted portion represents the deflection due to bending alone while the solid line represents the combined deflection due to bending and shear. In accordance with the plastic hinge concept, at a load of 160 kips the beam would turn into a mechanism with a hinge at B and deflect without any increase in load. This is represented by the solid line in Fig. 19. However, to illustrate a point regarding shear deflection, strain hardening is considered, and the deflection is now represented by the dashed lines. At a load of 185 kips, the nominal shear stress in the portion AB is 17.4 ksi which indicates that shear yielding has taken place. The flexural deflection at B is 3.69 in. The shear deflection for the portion AB is computed to be 0.205 in. while that for BC is 0.072 in. This immediately indicates a lack of closure of 0.133 in. as is shown directly below the beam sketch in Fig. 19. Since the shear angle change at B is fixed and the deflections must close out, the triangle is rotated to effect the closure, which reduces the shear deflection at B to 0.178 in. The total deflection at B is then 3.69 in. plus 0.18 in., or 3.87 in. total.

The physical significance of this lack of closure and necessary adjustment of the shear deflection can be seen by realizing that the yielded portion AB wanted to deflect the full amount while the elastic portion could not and thereby held it back to some extent. This action in turn caused a slight readjustment of the slopes at the ends of the beam which can be calculated. Obviously the shear yielding of AB could have been induced while the flexural deflection was in the elastic range had the load been closer to the end of the beam.

b. Beam Fixed at A and C (Fig. 20)

This beam is twice indeterminate. In this case in addition to the equations of equilibrium two relationships involving the deformation of the beam must be utilized. The two conditions utilized are that the sum of the angle changes between A and C must be zero and that the deflection of one end with respect to the tangent at the other must equal zero.

These conditions yield the following relationships for the elastic case:

$$M_A = \frac{Pab^2}{L^2}$$

$$M_C = \frac{Pa^2b}{L^2}$$

The dotted lines in Fig. 20 show the behavior of the beam when bending alone is considered. Initially the maximum moment occurs at A and the first hinge forms at this point when the fully plastic moment value of 6150 kip in. is reached. The load corresponding to the formation of this first hinge is 200 kips and at this load there are distinct breaks in the load-moment relationships as well as in the load-deflection relationship. As the load is increased the next hinge forms at B at a load of 309 kips. The maximum load of 320 kips is reached when the hinge moment is reached at point C and at this load the beam theo-

retically turns into a simple mechanism which deflects without any increase in load. It should be noted that the load-deflection curve has the two distinct breaks which are characteristic of continuous beams when bending alone is considered.

For bending and shear the expressions for the moments become only slightly more complicated. In this case the angle change expression is not altered since the algebraic sum of the shear angle changes will always be zero if statics is satisfied. However, there are added terms in the deflection relationship.

The two expressions are:

$$\frac{1}{EI}\left[\frac{Pab}{2} - \frac{M_A L}{2} - \frac{M_C L}{2}\right] = 0$$

$$\frac{1}{EI}\left[\frac{-Pab}{2}\left(\frac{L+b}{3}\right) + \frac{M_A L^2}{3} + \frac{M_C L^2}{6}\right] + \gamma_{AB}\, a - \gamma_{BC}\, b = 0$$

For the elastic case these reduce to:

$$M_A = \frac{Pab}{L} - M_C$$

$$M_C = \frac{\frac{Pa^2b}{6} + \frac{Pab}{KL}}{\frac{L^2}{6} + \frac{2}{K}}$$

where

$$K = \frac{A_w G}{EI}$$

As a check we note that if $a = b = \frac{L}{2}$ (the load is in the center of the beam) then $M_A = M_B = M_C = \frac{PL}{8}$, which is the same expression as for bending alone.

The solid lines in Fig. 20 show the behavior of the beam when bending combined with shear is considered. Just as in the case of bending alone (dotted lines in Fig. 20) the moment at A is initially a maximum. The inclusion of the shear terms in the foregoing relationship causes a slight readjustment of moment in the elastic range. At a load of 158 kips, before the hinge moment is attained at A, general shear yielding occurs in section AB which in turn continually transfers a larger proportion of the total load to section BC. As the load is increased there is a marked readjustment of moment at all three points with the moment at A decreasing slightly. The hinge moment is reached first at point C at a load of 271 kips. The next hinge forms at B at a load of 284 kips and the maximum load of 320 kips is attained when the last hinge forms at point A.

The equations given with the example are for the initial response only and have to be altered as the hinges form or the sections yield in shear. For many obvious reasons it would not be expected that the results of these analyses could be exactly duplicated experimentally. The important point to be drawn from the examples is that for cases of unsymmetrical loading, high shear forces can change the values of moment and slope from those which would be expected from flexure alone. This fact must be kept in mind by anyone planning tests and performing analyses where shear forces are high enough to be of importance.

General Observations

There are many limitations in utilizing the reserve plastic strength, two

of the more serious ones being local flange and web buckling. Flange or web buckling may seriously impair the rotation capacity of the member. Thick flanges and webs tend to cut down the possibility of buckling and this probably will be the trend where large rotation capacities are desirable. Thick flanges will provide greater moment capacity, especially as the strain hardening capacity is utilized. Larger moments may imply higher shear forces. Thus the possibility of shear yielding in the web and its corresponding effect on the behavior of the structure must be kept in mind when desirable section properties are studied. Also, the strengthening of beams by adding cover plates will often require that shear be taken into consideration.

As a general rule shear will be of primary importance in only unusual loading conditions. The most obvious example is a concentrated load near the end of a beam as might occur in the use of offset columns in buildings. Since general shear yielding in the webs will cause excessive deflections, the recommended practice is to avoid high shear when possible. However, when high shear forces are encountered, the data presented in this report will permit an estimate of the shear deflection to be made.

Shear deflections in the elastic range are quite often of importance. It is generally accepted that shear deflections should be considered when the span-depth ratio for simply supported beams is eight or less. For a 12WF120 beam section, simply supported and with a span eight times the depth, the elastic shear deflection represents about 25 per cent of the total deflection. The ratio of total span length to depth for which shear is of importance in plastic design is variable and depends on the boundary conditions and moment gradient. For a 12WF120 beam fixed at both ends and loaded at the center, general shear yielding in the web will occur at or before the yield moment is reached at the support or load point (both moments are equal in this case) for a span-depth ratio of about 11. This emphasizes the fact that the span-depth ratio need not be extremely small for shear deformation to be of importance.

SUMMARY

As far as is known from the literature, the tests described in this paper represent the first large-scale tests of continuous beams loaded far into the plastic range in which extremely high shear forces are present. On the basis of these tests it is concluded that no measurable reduction in the moment capacity (for the section and span used) is indicated.

These tests also provide data which make it possible to predict the components of deflection due to shear when regions of the beam have undergone general shear yielding. When compared with the current design specifications, portions of the test results indicate that where large shear forces are present the factors of safety may be somewhat low. This would seem to be particularly true for loadings of the type used in these tests. These are admittedly unusual, but they are not covered properly by the specifications.

Theoretical examples are presented which show that in unsymmetrically loaded beams when shear is considered the moment and slope values are different from those computed on the basis of flexure alone. This could be of major importance when shear yielding of the web occurs.

ACKNOWLEDGEMENT

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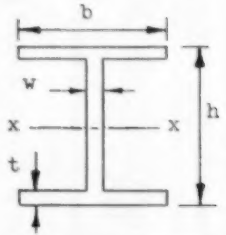
TABLE 1 SECTION PROPERTIES OF BEAM TEST SPECIMENS

Beams B1 and B2
8WF58 As-rolled Sections



Beam	Depth h in.	Web Thickness w in.	Flange Width b in.	Flange Thickness t in.	Area in^2	I_x $in.^4$	Z_x $in.^3$
Measured Values*							
B1	8.85	.518	8.132	.840	17.50	237	61.80
B2	8.82	.519	8.136	.824	17.25	232	60.68
AISC Handbook Values							
--	8.75	.510	8.222	.808	17.06	227.3	--

*Average of values taken at two locations on beam.

TABLE 2 TENSILE TEST COUPON DATA

Property*	Beam B1	Beam B2
Yield Stress (σ_{yp}) ksi	33.8	35.5
Youngs Modulus (E) ksi x 10^{-3}	31.8	31.5
Strain Corresponding to Beginning of Strain Hardening Region (ϵ_{sh}) in/in	0.012	0.012
Maximum Stress (σ_{max}) ksi	60.8	62.8
Per Cent Elongation (8 in. gage length)	33	32

*Based on nine coupons taken from one section (3 from each flange and 3 from the web). Values listed are average weighted values. The individual coupon data were weighted for (1) area represented and (2) distance from neutral axis.

TABLE 3 COMPARISON OF SHEAR STRESS VALUES IN THE ELASTIC RANGE

Beam B1

Load Kips	Shear** Kips	τ_{xy} from Rosettes* No. 2	No. 4	No. 5	$\frac{VQ}{Ib}$	$\frac{V}{A_w}$	τ_{xy} from Rosettes* No. 1	No. 3	$\frac{VQ}{Ib}$
17.5	9.3	2,550	2,880	2,670	2,340	2,240	2,550	2,730	2,290
40	20.9	6,030	5,780	6,030	5,250	5,040	6,100	5,710	5,150
50.5	26.3	8,430	7,260	7,900	6,600	6,350	7,810	8,560	6,370
59.5	31.2	10,720	8,410	9,260	7,830	7,500	9,310	16,670	7,680
66.5	34.5	12,650	9,400	10,660	8,650	8,300	11,590	----	8,500
81.5	42.1	14,890	11,450	13,440	10,500	10,150	15,370	----	10,400
100	51.4	----	13,870	----	12,900	12,400	----	----	----
113	57.6	----	15,530	----	14,400	13,900	----	----	----

Beam B2a

Load Kips	Shear** Kips	τ_{xy} from Rosettes* No. 7	No. 9	$\frac{VQ}{Ib}$	$\frac{V}{A_w}$	τ_{xy} from Rosettes* No. 6	No. 8	$\frac{VQ}{Ib}$
30	14.4	3,570	2,150	3,630	3,500	3,720	3,630	3,540
65.5	32.8	8,310	11,560	8,260	7,900	11,820	8,470	8,070
100	49.9	14,060	----	12,580	12,000	----	15,200	12,300
130	65.8	16,540	----	16,600	15,900	----	----	16,200

*For gage numbers and locations see Fig. 17

**These shears are slightly different from one-half the load because of slight dissymmetry in the position of loading and in the beam properties.

Note: All shear stress values are in units of psi.